SETON HALL UNIVERSITY

TWENTYFIRST ANNUAL

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MATHEMATICS COMPETITION

1. A car rental company charges \$36 a day and 22¢ a mile for renting a car. June rented a car for 5 days and was charged \$292.20. How many miles did she drive?

2. The degree measure of each angle in a regular octagon is how much larger than the degree measure of each angle in a regular hexagon?

3. Evaluate $a^{bc}+b^{ac}+c^{ab}$ if a = 4, b = -1, c = 1/2.

4. Jim drives a certain distance at an average rate of r miles per hour (where r is a positive real number). He then drives twice the original distance at an average rate of 2/3 r miles per hour. Find the average rate (in miles per hour) in terms of r for the total distance he drove.

5. Mixture A contains 45% alcohol and mixture B contains 75% alcohol. How many quarts of mixture A and mixture B must be combined to obtain 60 quarts of a mixture which contains 64% alcohol?

6. Simplify (where $t^2-2ct<0$): $\dfrac{(t-c)(c-t)}{\sqrt{2ct-t^2}}+\dfrac{c}{\sqrt{1-(t-c)^2/c^2}}+\sqrt{2ct-t^2}$.

7. Let n be a positive integer and define $F(n)$ to be the sum of the 2n smallest positive integral multiples of n. For example, $F(3)=1\cdot 3+2\cdot 3+3\cdot 3+4\cdot 3+5\cdot 3+6\cdot 3=21\cdot 3=63$. Find the largest prime number p for which $F(p)<15000$.

8. The real values of x for which $\dfrac{8x^3+48x^2+117x+108}{x^3+9x^2+27x+27}\le 4$ lie on an interval. Find the length of this interval.

9. One code (a “letter code”) consists of 3 letters; the letters can be chosen from A to Z, but the third cannot be an O or an I; a letter can appear at most twice and repeated letters must be in adjacent positions. A second code (a “digit code”) consists of 4 digits; the digits can be chosen from 0 to 9, but the first cannot be a 0 or a 1; a digit can appear at most twice and repeated digits must be in adjacent positions; and two pairs of double digits are allowed. By how much does the possible number of “letter” codes exceed the possible number of “digit” codes?

10. An ellipse lies on a coordinate plane and passes through the origin and the points (0,16) and (-4,0). If the major axis of the ellipse is parallel to the *y*-axis and is 34 feet long, find the length of the minor axis of the ellipse.

11. The real value of x for which $2^{4x+3} \cdot 3^{-3x+2} = 4^{2x+2} \cdot 5^{-x+1}$ can be written in the form $x = \dfrac{\log R}{\log T}$, where R and T are rational numbers. Find R and T (in simplest rational form).

12. The Lodi Loops are to play the Bogata Bouncers in a best of 5 series. The first two games are to be played in Lodi, the next two in Bogata (if a fourth is needed), and the fifth in Lodi (if needed). The probability that Lodi will win at home is 2/3 (and that Bogata will win at Lodi is 1/3); the probability that Lodi will win at Bogata is 1/2 if they have won at least one previous game and is 3/8 if they have won no previous games. Find the probability that Bogata wins the series in fewer than 5 games.

13. A toy store manager purchased a total of 43 items including blocks (at \$8.50 each), dolls (at \$12.20 each), trucks (at \$10.40 each), and puzzles (at \$6.40 each), for a total of \$410.00. The amount spent for the trucks and puzzles exceeded the amount spent for the blocks and dolls by \$30. The number of blocks and trucks purchased was three more than the number of dolls and puzzles purchased. How much was spent on the trucks?

14. Find the sum of the squares of the three values of x for which $\left(x^3 - x - 5\right)^3 - 3\left(x^3 - x - 5\right)^2 + 3\left(x^3 - x - 5\right) - 1 = 0.$

15. Triangle *ABC* has sides *AB*, *AC*, *BC* of lengths 24 feet, 40 feet, 56 feet respectively. Line segment *AD* is drawn from *A* to point *D* on side *BC*, and *AD* bisects angle *BAC*. Find the length of line segment *AD*.

16. Let *A* and *N* be positive real integers. Bob earns 2*A* dollars the first week, *A*/4 more than that the second week, and so forth, each successive week earning *A*/4 more than the previous week. Ron earns *A* dollars the first week, *A*/2 more than that the second week, and so forth, each successive week earning *A*/2 more than the previous week. After both have worked for *N* weeks, Ron has earned \$252 more than Bob. The sum of Bob’s earnings in the third week and Ron’s earnings in the second from last week was \$100. How much did Bob earn in *N* weeks?